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Elastic and inelastic scattering of electrons by a standing wave of intense and coherent light

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Abstract. The elastic and inelastic scattering of electrons by a standing wave of intense and coherent light are investigated by means of the Helmholtz–Kirchhoff diffraction theory. This approach has many features in common with the theory of Raman and Nath on the Debye–Sears effect. The time-averaged theories, used previously for the description of Kapitza–Dirac scattering, are shown to be inadequate.

1. Introduction

With the advent of powerful lasers the theoretical and experimental investigation of the elastic scattering of electrons by a standing light wave, known as the Kapitza–Dirac effect (Kapitza and Dirac 1933), has again become of interest (Schwarz 1973). The paper of Schwarz contains a critical survey of the available experimental evidence for the existence of this effect. In three recent communications (Ehlotzky 1974, 1975, Ehlotzky and Leubner 1974) we have shown that, despite its apparent simplicity, the theory of phase gratings in the Fraunhofer approximation of the Helmholtz–Kirchhoff diffraction theory may be successfully applied to the investigation of the above problem. This can be done after an appropriate expression has been derived for the refractive index of electrons traversing a standing wave of intense and coherent light. The resultant theory, which is extended in the present paper to the inclusion of inelastic scattering phenomena into our discussion, has many similarities with the corresponding investigations of Raman and Nath (1936) on the Debye–Sears effect (Debye and Sears 1932). The latter theory has been astonishingly successful in the proper interpretation of the experimental data, in particular of the Bär interference phenomena (Bär 1933), of which analogues will also be met in our present considerations.

2. Quasi-stationary theory of electron scattering by a standing light wave

For what follows we refer readers to our previous investigations (Ehlotzky 1974, 1975, Ehlotzky and Leubner 1974), in particular concerning notation and justification of various approximations. In accordance with Raman and Nath (1936), we shall start with the derivation of an approximate expression for the index of refraction of electrons passing a standing light wave. As has been done in most previous investigations (Schwarz 1973, Ehlotzky 1974, 1975, Ehlotzky and Leubner 1974), we shall describe the standing light wave by a classical monochromatic electromagnetic background field of frequency

ω and wavenumber $k = 2\pi/\lambda$, having its direction of propagation along the ξ axis and its vector $\boldsymbol{\epsilon}$ of linear polarization perpendicular to the plane of scattering. Furthermore, we shall assume that the motion of the electrons in the standing light wave may be described by the non-relativistic Schrödinger equation, since, after all, only sufficiently slowly moving electrons will lead to measurably large, though actually still very small, scattering angles. Hence, the electron beam may be supposed to impinge on the standing light wave close to the normal on the axis ξ. Taking this normal as the z direction the electrons are incident at small angles θ_0 with respect to this axis and leave the standing wave at similarly small angles θ. Consequently, putting $\hbar = c = 1$, the Schrödinger equation will read

$$\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m} - \frac{e^2}{2m}A^2\right)\Psi = 0 \tag{1}$$

where, as usual, we have taken the radiation gauge with $\nabla A = 0$ and the term $\boldsymbol{A}\cdot\nabla$ has dropped out on account of our choice of the scattering plane. At present, by the way, it appears that the choice of an arbitrary scattering plane would, on account of the term $\boldsymbol{A}\cdot\nabla$, only lead to unnecessary additional complications†. The vector potential $\boldsymbol{A}$ of the standing light wave is given by (Ehlotzky and Leubner 1974)

$$\boldsymbol{A} = 2A_0\boldsymbol{\epsilon}\cos\omega t\cos k\xi \tag{2}$$

assuming 100% reflection of the ingoing laser beam at a plane mirror placed perpendicular to the direction of incidence of the laser light.

Inserting (2) into (1) we may conveniently write

$$(\nabla^2 + 2\mathrm{i}m\partial_t - e^2A_0^2)\Psi = \{e^2A_0^2[\cos 2\omega t + \cos 2k\xi(1+\cos 2\omega t)]\}\Psi \tag{3}$$

and we shall first solve the truncated equation, which is obtained by putting the left-hand side equal to zero. This yields $\Psi_0 = \exp[-\mathrm{i}(E_0t - \boldsymbol{K}\cdot\boldsymbol{x})]$ where we assume the ingoing electrons to have kinetic energy $E_0 = p_0^2/2m$ and wavenumber $K_0 = 2\pi/\Lambda_0$ so that $K = K_0n_0$ is the wavenumber in the averaged field, the refractive index of which is given by $n_0 = [1-(eA_0/K_0)^2]^{1/2}$. For the solution of the exact equation (3) we shall now try the corresponding *ansatz* $\Psi = u(\boldsymbol{x}, t)\exp(-\mathrm{i}E_0t)$ and since in all relevant experiments we shall always have $\hbar\omega \ll E_0 \ll mc^2$ it is reasonable to assume $u(\boldsymbol{x}, t)$ to be a comparatively slowly varying function of time so that we will be permitted to make the following quasi-stationary approximation $\mathrm{i}\partial_t\Psi \simeq E_0\Psi$ by means of which we get from (3)

$$(\nabla^2 + K^2n^2(\xi, t))u(\boldsymbol{x}, t) = 0 \tag{4}$$

and the refractive index of our grating, which is represented by the standing light wave, will be given by

$$n(\xi, t) = \{1-(eA_0/K)^2[\cos 2\omega t + \cos 2k\xi(1+\cos 2\omega t)]\}^{1/2}. \tag{5}$$

Similarly, since $k \ll K$, we may also assume $u(\boldsymbol{x}, t)$ to be slowly varying in $\boldsymbol{x}$, from which we obtain the approximate solution of (3) and (4) in the form

$$\Psi(\boldsymbol{x}, t) \simeq \exp[-\mathrm{i}(E_0t - n(\xi, t)\boldsymbol{K}\cdot\boldsymbol{x})]. \tag{6}$$

† Our choice of the scattering plane has been suggested to us by the work of Gush and Gush (1971). Furthermore, for the induced scattering processes considered here any momentum transfer will always be in the direction ξ and therefore perpendicular to $\boldsymbol{\epsilon}$.

But, as we have learned in our earlier investigations (Ehlotzky and Leubner 1974), this approximation is consistent only if we require $kd(2\mu/\beta)^2 \ll 1$ from which it follows necessarily that $eA_0/K = (2/n_0)(\mu/\beta) \ll 1$. Here we have introduced the diameter d of the laser beam, the electron velocity β measured in units of c and the Kibble parameter μ†. Hence we may use the further approximations $n_0 \simeq 1$, $\boldsymbol{K} \simeq \boldsymbol{K}_0$ and

$$n(\xi, t) \simeq 1-2(\mu/\beta)^2[\cos 2\omega t+\cos 2k\xi(1+\cos 2\omega t)] \tag{7}$$

so that finally our wavefunction (6) assumes the form

$$\Psi(\boldsymbol{x}, t) \simeq \exp[-\mathrm{i}(E_0 t-\boldsymbol{K}_0 . \boldsymbol{x})] \exp\{-\mathrm{i}2K_0 z(\mu/\beta)^2[\cos 2\omega t+\cos 2k\xi(1+\cos 2\omega t)]\}. \tag{8}$$

In the second exponential of this expression we have made use of our assumption that the electron beam impinges on the standing light wave very close to the z direction.

According to our previous investigations (Ehlotzky and Leubner 1974) we may neglect all sorts of 'edge effects' and consequently we may assume the electron beam to enter and to leave the standing light wave fairly abruptly. If, therefore, the electron wave (8) has traversed the distance $z = d$ in the standing light wave, we may define a time-dependent transmission function $D(\xi, t)$ of our grating (Born and Wolf 1964) of length $D = N\lambda/2$, corresponding to the diameter of the electron beam (Ehlotzky and Leubner 1974), where N is a very large positive and integral number representing, so to speak, the number of bars of the grid. Thus we have

$$D(\xi, t) = \mathrm{e}^{-\mathrm{i}E_0 t} \exp\{-\mathrm{i}\rho[\cos 2\omega t+\cos 2k\xi(1+\cos 2\omega t)]\} \tag{9}$$

where the factor $\exp(-\mathrm{i}E_0 t)$ has only been retained for later convenience of interpretation. $\rho = 2K_0 d(\mu/\beta)^2$ is the most important parameter of our theory and it can be quite easily shown (Fedorov 1967, Gush and Gush 1971) that $\rho = (4\pi r_0/\omega^2)I_0 T$ where I_0 is the intensity of the laser beam and T the time taken for the electrons to pass through the standing light wave. Since, however, we had to assume for the validity of the above approximate transmission function that $kd(2\mu/\beta)^2 \ll 1$, the condition

$$\rho = 2K_0 d(\mu/\beta)^2 \ll K_0/2k = \lambda/2\Lambda_0$$

follows immediately. But, for electron energies of about 100 eV and for laser beam wavelengths $\lambda \simeq 10^{-4}$ cm usually considered, we shall certainly have $\lambda/2\Lambda_0 \gg 1$. Consequently, in our discussions later on, we will be permitted to consider small as well as intermediate values of ρ. This corresponds, according to our above conditions, to still relatively small values of the Kibble parameter, namely $\mu \simeq 10^{-6}$, being equivalent to a power output of the laser of about 10^7 W cm^{-2} (Schwarz 1973, Sarachik and Schappert 1970).

Furthermore, we can infer from (9) that in our approximation the standing light wave acts on the transmitted electrons as a phase grating, which is characterized by $|D(\xi, t)| = 1$. This grating is periodic in space with period $\lambda/2$ and in time with period $\tau/2$, where $\tau = 2\pi/\omega$. It is important to realize in this connection that the periodic structure of $D(\xi, t)$ will be preserved, even if a more precise solution of the Schrödinger equation (3) can be obtained, from which the dissipative character of the inelastic scattering components, concomitant with the elastic Kapitza–Dirac effect, becomes apparent. In that case we shall obtain a more general transmission function $D(\xi, t) = A(\xi, t) \exp(\mathrm{i}\Phi(\xi, t))$, where A and Φ are periodic in ξ and t with the above

† This parameter is defined by $\mu^2 = (eA_0/2m)^2 = (2\pi/m)(r_0 I_0/\omega^2)$, where $r_0 = e^2/m$ and $I_0 = \omega^2 A_0^2/8\pi$ using $\hbar = c = 1$.

periods. Hence the principal structure of the diffraction pattern, which we shall discuss below, will not be changed in going to higher laser beam intensities than those permitted within the framework of our approximation.

Introducing (9) into the Fraunhofer diffraction integral (Born and Wolf 1964), for which the conditions of validity are certainly fulfilled (Ehlotzky and Leubner 1974, Ehlotzky 1975), we obtain for the diffracted electron wave

$$\Psi_D(\alpha, t) = C \exp[-i(E_0 t + \rho \cos 2\omega t)] \times \int_0^D \exp\{-i[K_0(\alpha - \alpha_0)\xi + \rho(1 + \cos 2\omega t)\cos 2k\xi]\}\, d\xi \tag{10}$$

where $\alpha = \sin\theta$ and $\alpha_0 = -\sin\theta_0$. Since, however, $\cos 2k\xi$ has period $\lambda/2 = \pi/k$ we may put $\xi = \xi_0 + \nu\pi/k$ and therefore (10) reduces to

$$\Psi_D(\alpha, t) = \exp[-i(N-1)\gamma]\frac{\sin N\gamma}{\sin\gamma}\exp[-i(E_0 t + \rho\cos 2\omega t)]\Psi_{\lambda/2}(\alpha, t) \tag{11}$$

where

$$\Psi_{\lambda/2}(\alpha, t) = \int_0^{\lambda/2} \exp\{-i[K_0(\alpha-\alpha_0)\xi_0 + \rho(1+\cos 2\omega t)\cos 2k\xi_0]\}\, d\xi_0$$

$$= \tfrac{1}{2}\lambda \sum_{s=-\infty}^{+\infty} J_s[\rho(1+\cos 2\omega t)](-i)^s e^{-i(\gamma+s\pi)}\frac{\sin(\gamma+s\pi)}{\gamma+s\pi}. \tag{12}$$

The latter expression has been obtained by realizing that the exponential under the integral sign represents the generating function of Bessel functions. We have introduced the abbreviation $\gamma = K_0(\alpha-\alpha_0)\lambda/4$.

From (11) we conclude immediately that the principal maxima of the Fraunhofer diffraction pattern are determined by (Ehlotzky and Leubner 1974, Ehlotzky 1975, Born and Wolf 1964)

$$\gamma = n\pi, \qquad n = 0, \pm 1, \pm 2, \ldots$$

or

$$p_0(\sin\theta + \sin\theta_0) = 2n\hbar k. \tag{13}$$

As we have done in our previous work (Ehlotzky and Leubner 1974, Ehlotzky 1974, 1975), we shall confine ourselves in the following primarily to these diffraction maxima and, with (13) being fulfilled, we conclude at once from (12) that the only surviving term of the infinite sum over s will be the one for which $s = -n$. Consequently, if we Fourier-decompose this term with respect to time t, we get

$$\Psi_D(\alpha_n, t) = (CN\lambda/2)(-1)^{Nn} i^n \exp[-i(E_0 t + \rho\cos 2\omega t)] J_n[\rho(1+\cos 2\omega t)]$$

$$= (CN\lambda/2)(-1)^{n(N-1)} e^{-iE_0 t} \sum_{\nu=-\infty}^{+\infty} f_{n,\nu}(\rho)\, e^{-i\nu 2\omega t} = \sum_{\nu=-\infty}^{+\infty} \Psi_{D,\nu}(\alpha_n, t) \tag{14}$$

where, putting $2\omega t = \phi$, the Fourier components $f_{n,\nu}(\rho)$ are given by

$$f_{n,\nu}(\rho) = \frac{(-i)^n}{2\pi}\int_0^{2\pi} \exp[i(\nu\phi - \rho\cos\phi)] J_n[\rho(1+\cos\phi)]\, d\phi$$

$$= (2\pi)^{-2}\int_0^{2\pi}\int_0^{2\pi} \exp(i\nu\phi + in\psi) \times \exp[-i\rho(\cos\phi + \cos\psi + \cos\phi\cos\psi)]\, d\phi\, d\psi \tag{15}$$

and in the second part of this equation use has been made of the integral representation of J_n.

Hence, according to (14), the electron wave scattered into the principal maximum of index n is composed of an infinite sum of incoherent components $\psi_{D,\nu}(\alpha_n, t)$ each corresponding to the scattering of electrons by the standing light wave on account of the induced emission or absorption of a net number of $2|\nu|$ photons into or from the standing laser field respectively and of simultaneous momentum change of amount (13). Obviously, the term $\nu = 0$ corresponds to elastic electron scattering and will therefore describe the ordinary Kapitza–Dirac effect, which has been discussed exclusively in earlier investigations (Kapitza and Dirac 1933, Schwarz 1973, Ehlotzky and Leubner 1974, Ehlotzky 1975). Our more general result (14) and (15) is analogous to the findings of Raman and Nath (1936) on the Debye–Sears effect (Debye and Sears 1932).

If we switch off the standing light wave, we easily conclude from the diffraction integral (10) that the amplitude of the ingoing electron wave is given by $\Psi_0(t) = (CN\lambda/2)\exp(-iE_0t)$ (Ehlotzky and Leubner 1974) and consequently we obtain for the probability of electron scattering into the nth diffraction maximum $|\Psi_D(\alpha_n, t)|^2/|\Psi_0(t)|^2$. Since, however, the time of observation is usually much larger than the period of the standing light wave we get for this probability after averaging over t

$$P_n(\rho) = \sum_{\nu=-\infty}^{+\infty} P_{n,\nu}(\rho), \qquad P_{n,\nu}(\rho) = |f_{n,\nu}(\rho)|^2 \tag{16}$$

as can be easily inferred from (14). The $P_{n,\nu}(\rho)$ in (16) are the probabilities of elastic and inelastic electron scattering into the diffraction maxima of order n with induced emission or absorption of $2|\nu|$ quanta from the standing light wave.

The remainder of our considerations will therefore be devoted to the evaluation of the probabilities $P_{n,\nu}(\rho)$ from (15) and to the discussion of their specific properties, which imply some important conclusions quite different from previous investigations.

Let us start with the enumeration of the general properties of the $P_{n,\nu}(\rho)$. First of all we infer from the definition (15) of the Fourier amplitudes $f_{n,\nu}(\rho)$ that

$$P_{n,\nu}(\rho) = P_{n,-\nu}(\rho) = P_{-n,\nu}(\rho) = P_{-n,-\nu}(\rho) = P_{\nu,n}(\rho). \tag{17}$$

Secondly, if we evaluate the total diffraction probability $P_n(\rho)$ for the maximum of order n from (15) and (16), thereby using the identity (Gel'fand and Shilov 1964)

$$\sum_{\nu=-\infty}^{+\infty} e^{-i\nu(\phi-\phi')} = 2\pi \sum_{l=-\infty}^{+\infty} \delta[\phi' - (\phi + 2\pi l)]$$

we get immediately

$$P_n(\rho) = (2\pi)^{-1} \int_0^{2\pi} J_n^2[\rho(2\cos^2\phi/2)]\, d\phi \tag{18}$$

and therefore the total probability of scattering into any of the infinitely many diffraction maxima will be given by

$$P = \sum_{n=-\infty}^{+\infty} P_n(\rho) = 1 \tag{19}$$

on account of the completeness relation of the Bessel functions. Hence the probabilities $P_{n,\nu}(\rho)$ fulfil the conditions of normalization and conservation of probability in our scattering problem. This result also shows that our approximation is consistent with the assumption of no electrons being reflected by the grating and of all scattering taking place into the principal maxima of the Fraunhofer diffraction pattern (Ehlotzky 1974).

If we exploit the periodicity of the integrand in (15), we can rewrite $f_{n,\nu}(\rho)$ as

$$f_{n,\nu}(\rho) = \pi^{-2} \exp(\mathrm{i}\rho) \int_0^\pi \mathrm{d}\phi \int_0^\pi \mathrm{d}\psi \cos \nu\phi \cos n\psi \exp(-4\mathrm{i}\rho \cos^2 \phi/2 \cos^2 \psi/2). \tag{20}$$

Expanding the exponential under the integral sign and integrating term by term we obtain (Gröbner and Hofreiter 1961, part 2, p 110, integral 14(*b*))

$$f_{n,\nu}(\rho) = \exp(\mathrm{i}\rho) \sum_{k=\max(n,\nu)} [(-\mathrm{i}\rho/4)^k/k!]\binom{2k}{k-\nu}\binom{2k}{k-n}. \tag{21}$$

By virtue of (17) it is no restriction to assume $n \geqslant \nu$. The scattering probabilities $P_{n,\nu}(\rho) = |f_{n,\nu}(\rho)|^2$ have been evaluated numerically from (21) for $n = 0, 1, 2, 3$ and $\nu = 0, 1$.

The total probability of scattering (elastic and inelastic) into the nth diffraction maximum, $P_n(\rho)$, is obtained from (18) by replacing J_n^2 by its power series (Abramowitz and Stegun 1970, p 360, equation (9.1.14)) and subsequent term by term integration to yield

$$P_n(\rho) = \sum_{k=0}^{\infty} (-1)^k(\rho/4)^{2n+2k}\{(4n+4k)!/(2n+k)![(n+k)!]^2(2n+2k)!\}. \tag{22}$$

These probabilities have been calculated for $n = 0, 1, 2, 3$ and are shown together with the probabilities $P_{n,\nu}(\rho)$ in figure 1. For comparison we also give the functions $\overline{P_n(\rho)} = \mathrm{J}_n^2(\rho)$, ie the scattering probabilities of the time-averaged theory (Schwarz 1973, Ehlotzky and Leubner 1974, Ehlotzky 1974, 1975, Fedorov 1967).

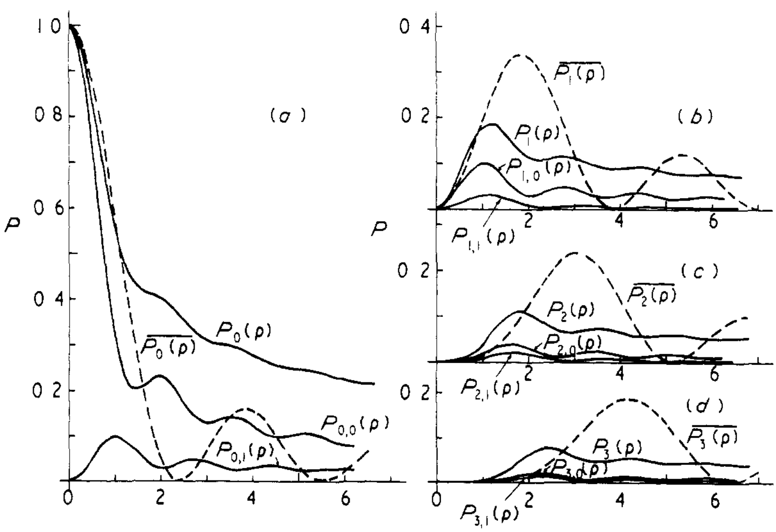

Figure 1. (*a*), (*b*), (*c*) and (*d*) show the probabilities $P_{n,\nu}(\rho)$ of elastic and inelastic electron scattering and the corresponding total probabilities $P_n(\rho)$ for $n = 0, 1, 2$ and 3 respectively. The broken lines, for comparison, represent the scattering probabilities $\overline{P_n(\rho)}$ of the time-averaged theory.

3. Conclusions

Summarizing, the most important conclusions that can be drawn from our above quasi-stationary theory of the elastic and inelastic scattering of electrons by a standing wave of intense and coherent light are the following.

(i) Contrary to the plausibility arguments presented in all previous investigations of this problem (Schwarz 1973, Ehlotzky and Leubner 1974, Ehlotzky 1974, 1975, Fedorov 1967, Gush and Gush 1971), the time-averaged theory only leads to fairly correct values of the scattering probabilities for sufficiently small values of ρ (ie $\rho \ll 1$). For increasing ρ, however, this theory by far overestimates the scattering probabilities for the diffraction maxima $n > 0$ as regards their order of magnitude as well as their oscillatory behaviour. The diagrams of figure 1 also tell us that 50% and more of the total diffraction probabilities $P_n(\rho)$ is due to inelastic electron scattering and for intermediate values of ρ only the lowest-order scattering probabilities $P_{n,\nu}(\rho)$ have appreciable values. This confirms our assumptions leading to our quasi-stationary approximation for it shows that the Fourier coefficients $f_{n,\nu}$ in the expansion (14), corresponding to 'slow' oscillations with time, dominate. It is convenient for order of magnitude estimates to write the parameter ρ in the form $\rho = (4\pi r_0 c/\hbar)(I_0 T/\omega^2) = 10^{26} I_0 T/\omega^2$, using cgs units (Ehlotzky and Leubner 1974, Fedorov 1967, Gush and Gush 1971). Taking for example $I_0 = 10^7\ \mathrm{W\,cm^{-2}}$, $T = 2\times10^{-9}\ \mathrm{s}$ and $\omega = 2\times10^{15}\ \mathrm{s^{-1}}$ (Schwarz 1973), we get $\rho = 5$.

(ii) In our quasi-stationary solution of the scattering problem, in which also the $\boldsymbol{p}\cdot\boldsymbol{A}$ part of the electromagnetic interaction has been eliminated by conveniently choosing the plane of scattering perpendicular to the vector of linear polarization of the standing light wave, the general features of the Fraunhofer diffraction pattern remain the same for the elastic as well as inelastic processes, if compared with the simplified time-averaged theory discussed earlier (Schwarz 1973, Ehlotzky and Leubner 1974, Ehlotzky 1974, 1975, Fedorov 1967).

(iii) For every order n of the diffraction pattern the scattered wave $\Psi_D(\alpha_n, t)$ is, according to (14), the incoherent sum of waves $\Psi_{D,\nu}(\alpha_n, t)$ corresponding to electrons of energy $E_0+2\omega\nu$. Hence, for a particular diffraction maximum n the total scattering probability $P_n(\rho)$ will be given by (16) and (18). On the other hand, for fixed value of the index ν all diffracted waves $\Psi_{D,\nu}(\alpha_n, t)$ are coherent to one another and may therefore interfere. This possibility of interference of the, in general, inelastically scattered electrons has its analogue in the diffraction of light by a standing wave of ultrasound. Here the corresponding interference phenomena have been predicted many years ago by Raman and Nath (1935, 1936) and have been observed by Bär (1933). The above properties of interference show that, even in the case of inelastic electron scattering, the standing light wave preserves, at least within the framework of our approximations, its character as a phase grating thus making it still feasible to observe Kapitza–Dirac scattering (now including inelastic effects) by means of the phase contrast method, as has been suggested by us earlier (Ehlotzky and Leubner 1974).

(iv) Since for every diffraction maximum of order n we shall have the same energy spectrum of scattered electrons $E_0+2\nu\omega$, one should be able to observe this energy splitting in close to the forward direction, even if the diffraction pattern itself can not be resolved. The only condition to be fulfilled for that purpose is $\Delta E_0 \ll 2\omega$, ie high monochromaticity of the electron beam, which is used for the experiment. For example, if ω is of the order of a few electron volts ΔE_0 will have to be about 0·1 eV.

(v) The general forms of the refractive index $n(\xi, t)$ of the standing light wave and of the wavefunction $\Psi(\boldsymbol{x}, t)$ of electrons traversing this wave, which we have derived in (5) and (6), show that at the antinodes of our grating the standing light wave may become totally reflective for electrons as soon as $(eA_0/K)^2 \gtrsim 1$, even if time-dependent effects are permitted to take place. Consequently, for sufficiently high laser beam intensities backward scattering of electrons should be observed in such experiments. This effect, therefore, is definitely not the result of the time-averaging approximation used earlier (Ehlotzky 1975). The detailed evaluation of the reflection probabilities for the standing light wave acting as a reflection grating is a rather complicated matter, for at high laser beam intensities our approximations, on which our present calculations are based, will break down and much more exact solutions of the Schrödinger equation (3) will be required.

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References

Abramowitz M and Stegun I A 1970 *Handbook of Mathematical Functions* (Washington: National Bureau of Standards)
Bär R 1933 *Helv. Phys. Acta* **8** 591
Born M and Wolf E 1964 *Principles of Optics* 2nd edn (Oxford: Pergamon Press) chap 8.6
Debye P and Sears F W 1932 *Proc. Natl Acad. Sci.* **18** 409
Ehlotzky F 1974 *Phys. Lett.* **47A** 491–2
—— 1975 *Can. J. Phys.* **53** 157–64
Ehlotzky F and Leubner C 1974 *Opt. Commun.* **10** 175–80
Fedorov M V 1967 *Sov. Phys.–JETP* **25** 952–9
Gel'fand I M and Shilov G E 1964 *Generalized Functions*, vol 1 (New York: Academic Press) p 31
Gröbner W and Hofreiter N 1961 *Integraltafeln* (Vienna: Springer-Verlag)
Gush R and Gush H P 1971 *Phys. Rev.* D **3** 1712–21
Kapitza P L and Dirac P A M 1933 *Proc. Camb. Phil. Soc.* **29** 297–300
Raman C V and Nath N S N 1935 *Proc. Ind. Acad. Sci.* A **2** 406–20
—— 1936 *Proc. Ind. Acad. Sci.* A **3** 75–84, 119–25
Sarachik E S and Schappert G T 1970 *Phys. Rev.* D **1** 2738–53
Schwarz H 1973 *Phys. Lett.* **43A** 457–8